Adjoint based shape perturbations for control of stability derivatives using kinetic meshfree method

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ABSTRACT

This paper demonstrates the use of adjoint based shape sensitivities to make incremental changes in stability derivatives. For this purpose, the primal, tangent linear and adjoint meshfree least squares kinetic upwind method (LSKUM) based solvers for $2D$ inviscid compressible flows are employed. The tangent linear and adjoint LSKUM solvers are constructed using algorithmic differentiation techniques. Here, the tangent solver computes the stability derivative and the adjoint solver yields the shape sensitivites of stability derivative. Numerical results are shown for the MS0313 airfoil to make incremental changes in the stability derivatives through shape perturbations.

Keywords: Stability derivatives, shape sensitivites, discrete adjoint method, Meshfree LSKUM.

1 Introduction

Shape sensitivities of various stability derivatives are required in optimal aerodynamic design. They are required in aerodynamic shape optimisation of full aircraft as there is a coupling between aerodynamic efficiency and stability derivatives [1]; in this work, stability derivatives are constraints. Shape sensitivities are required for making incremental changes in the stability derivatives (which are important inputs to control law design) by perturbing shapes. Further, they are also required in the design of high lift configuration (HLD) where there is a need to shift the $C_L(\alpha)$ curve, increase the slope of $C_L(\alpha)$ and change the shape to delay the stall so that higher C_{Lmax} can be obtained. Hence computation of stability derivatives and their shape sensitivities are essential for optimal design. It is possible to perform shape optimisation by imposing constraints on stability derivatives (such as C_{L_α}, C_{m_α}, C_{N_β} etc.) or directly using them as objective functions. In this research an attempt has been made to obtain incremental changes in C_{m_α} through shape perturbations.

2 An adjoint approach for the shape sensitivities of stability derivatives

Consider the problem of finding the perturbed shapes that control (increase or decrease) the longitudinal static stability derivative C_{m_α} in inviscid compressible flows. It amounts to a PDE constrained optimisation problem. In the discrete form, the optimisation problem can be formulated as

$$\max_{\boldsymbol{X}}/\min \quad J(\boldsymbol{U},\alpha,\boldsymbol{X}) = \frac{dC_m}{d\alpha}, \quad \text{subject to} \quad \boldsymbol{U} = G(\boldsymbol{U},\alpha,\boldsymbol{X}) \tag{1}$$

Here, J is the scalar objective function, which is a stability derivative. G represents a fixed point iteration of the primal meshfree solver based on LSKUM [2] for the numerical solution of $2D$ Euler equations. $\boldsymbol{U}$ is the converged flow solution, α is the angle of attack and $\boldsymbol{X}$ is the control vector. Assuming free node parametric representation of the shape, the control vector $\boldsymbol{X}$ consists of the shape coordinates (x, y) that define the geometry of the $2D$ configuration of interest.

In general, the shape sensitivities of a stability derivative can be evaluated in a two-step procedure. In the first step, the objective function, which is a stability derivative, is computed using the tangent linear solver. In the next step, an adjoint solver is constructed over the tangent solver to obtain the shape sensitivities. In the present work, the tangent linear LSKUM solver is constructed by algorithmically differentiating (AD) [3] the primal LSKUM solver. On the other hand, the adjoint LSKUM solver is constructed by differentiating the tangent linear LSKUM solver. In summary, we require three solvers, namely, the primal, tangent linear and adjoint LSKUM solvers. Typically, the run-time of the tangent linear LSKUM solver is around a factor of 3 compared to the primal LSKUM solver, while the run-time of the adjoint LSKUM solver that computes the shape sensitivities of stability derivatives is around a factor of 15 compared to the primal solver.

The point-wise shape sensitivities obtained from the adjoint solver are non-smooth and contain high frequency oscillations. In order to ensure that the perturbed shapes obtained at every optimisation cycle remain smooth, the shape sensitivities must be smoothed. In the present work, the sensitivities are smoothed using a two step procedure. In the first step, the sensitivities are smoothed using Sobolev gradient smoothing [4, 5]. Our investigations have shown that the direct use of these sensitivities with large step sizes in the gradient algorithm resulted in non-smooth shapes. To obtain smooth shapes, in the second step, the sensitivities are again smoothed using the Savitzky-Golay filter [6].

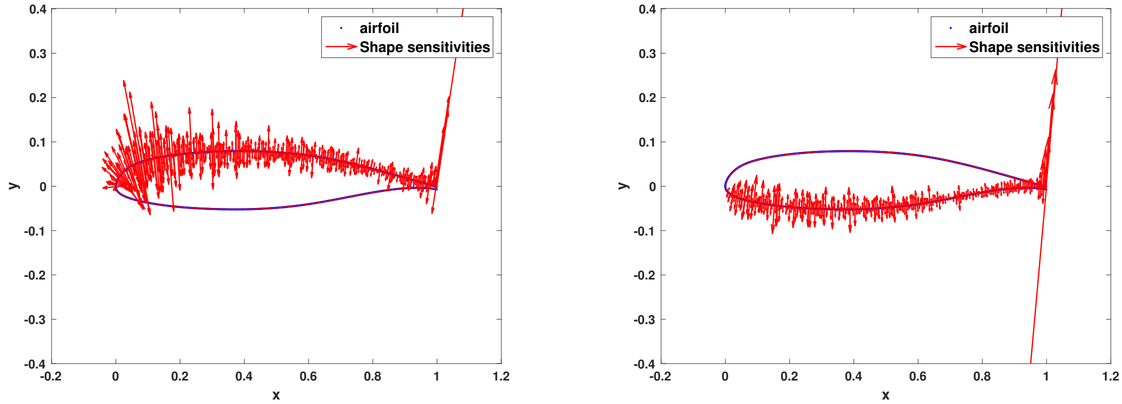
During the optimisation, as the shape changes, the interior points near the airfoil may move closer or farther to the airfoil geometry. It may also happen that the points fall inside the airfoil. As the interior points move very close to the airfoil, its least-squares matrices may become ill-conditioned, leading to inaccurate computation of spatial derivatives or code divergence. To prevent the ill-conditioning of the least-squares matrices and the movement of points inside the geometry, at every optimisation cycle the nearby interior points are moved according to the perturbation of their nearest shape points [5].

3 Results and discussion

The test case under investigation is the $MS0313$ airfoil with flow conditions corresponding to cruise at $M_\infty = 0.35$ and $\alpha = 4^0$. The computational domain consists of $58,519$ points, while the airfoil shape is discretised with 1282 points.

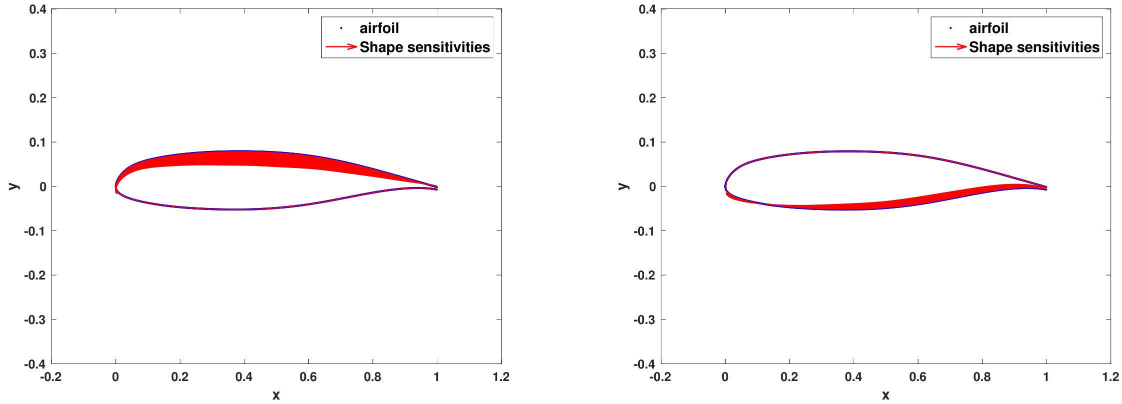
For the optimisation problem, the imposed geometric constraints are that the chord length and the trailing edge points of the airfoil are fixed. This implies that the y-coordinates of the rest of 1275 points on the airfoil form the control vector. Figure 1 shows the normal components of the shape sensitivities of $(dC_m/d\alpha)$. We observe that the sensitivities are non-smooth and highly oscillatory. Using these sensitivities in the gradient algorithm will lead to non-smooth and unphysical shapes. In order to get smoothed shapes, the raw sensitivities are smoothed using Sobolev gradient smoothing [4]. Figure 2 shows the smoothed sensitivities. These plots show that the sensitivities are dominant in the neighbourhood of mid-chord region of the airfoil on suction side. Depending on the choice of the objective function, the airfoil shape is perturbed in the appropriate direction of sensitivities.

Figure 3(a) shows that the slope of $C_m(\alpha)$ curve for the perturbed shape decreases after a few cycles of optimisation. That is, the pitching moment coefficient becomes more negative with the increase in the angle of attack. Figure 3(b) shows the $C_L(\alpha)$ curve for the baseline and perturbed shapes. Figure 4(a) shows that the slope of $C_m(\alpha)$ curve increases for the perturbed shape. Figure 4(b) shows the corresponding $C_L(\alpha)$ curve. Table 1 shows a comparison of the lift, pitching moment and longitudinal stability derivative $(dC_m/d\alpha)$ for the baseline and perturbed shapes at cruise conditions. Since we did not impose any constraints on C_L and C_m during the formulation of the optimisation problem, the force coefficients for the perturbed shapes got deviated from the baseline shape values.



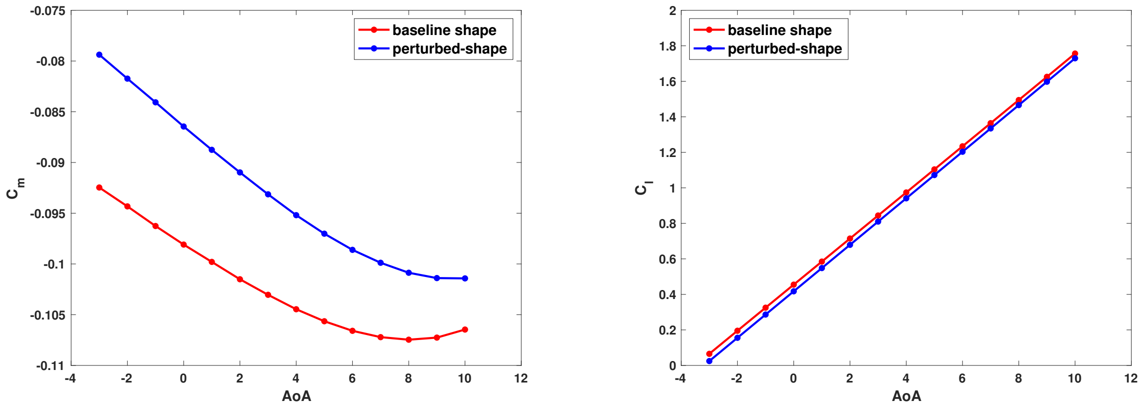
(a) Suction side sensitivities (b) pressure side sensitivities

Figure 1: Normal components of the shape sensitivities of $dC_m/d\alpha$ for the baseline airfoil.



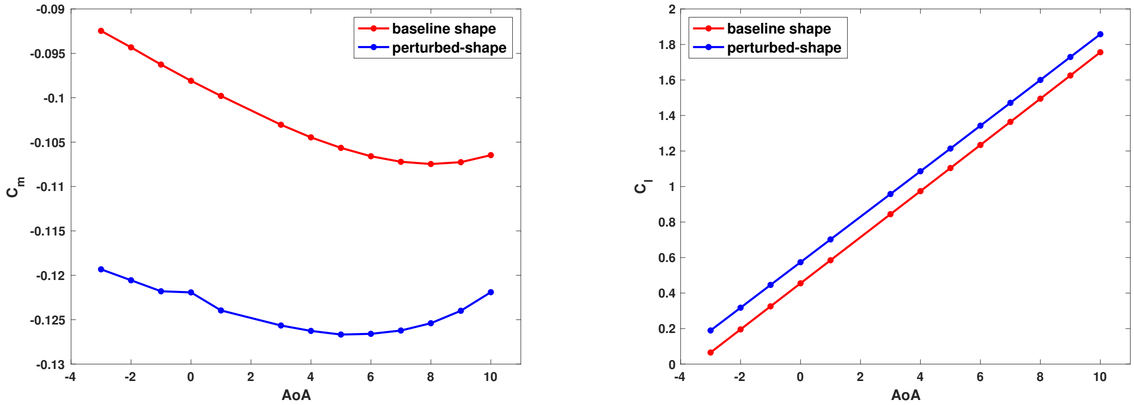
(a) Suction side sensitivities (b) pressure side sensitivities

Figure 2: Smoothed shape sensitivities of $dC_m/d\alpha$ for the baseline airfoil.



(a) $C_m(\alpha)$ curve (b) $C_L(\alpha)$ curve

Figure 3: Minimise $dC_m/d\alpha$: Comparison of $C_m(\alpha)$ and $C_L(\alpha)$ curves for the baseline and perturbed shapes.



(a) $C_m(\alpha)$ curve (b) $C_L(\alpha)$ curve

Figure 4: Maximise $dC_m/d\alpha$: Comparison of $C_m(\alpha)$ and $C_L(\alpha)$ curves for the baseline and perturbed shapes.

Airfoil shape	C_l	C_m	C_{m_α}
Baseline	0.9741	-0.1045	-1.2081×10^{-3}
Minimise C_{m_α}	0.9411	-9.5192×10^{-2}	-1.8888×10^{-3}
Maximise C_{m_α}	1.0860	-0.1263	-3.9181×10^{-4}

Table 1: Comparison of the lift, pitching moment, and longitudinal stability derivative for the baseline and perturbed shapes.

References

[1] Charles A. Mader and Joaquim R. R. A. Martins. Computing stability derivatives and their gradients for aerodynamic shape optimization. *AIAA Journal*, 52(11):2533–2546, 2014. URL `https://doi.org/10.2514/1.J052922`.

[2] S M. Deshpande, P S. Kulkarni, and A K. Ghosh. New developments in kinetic schemes. *Computers Math. Applic.*, 35(1):75–93, 1998.

[3] A. Griewank and A. Walther. *Evaluating derivatives: Principles and techniques of Algorithmic Differentiation*. SIAM, 2008.

[4] A. Jameson. Aerodynamic design via control theory. *J. Sci. Comput.*, 3:233–260, 1988.

[5] Keshav S Malagi, Nischay R. Mamidi, Anil Nemili, Vasudev Ramesh, and Suresh M Deshpande. Adjoint based aerodynamic shape optimisation using kinetic meshfree method. *International Journal of Computational Fluid Dynamics*, 2023. URL `https://doi.org/10.1080/10618562.2023.2230897`.

[6] Abraham. Savitzky and M. J. E. Golay. Smoothing and differentiation of data by simplified least squares procedures. *Analytical Chemistry*, 36(8):1627–1639, July 1964. doi: 10.1021/ac60214a047. URL `https://doi.org/10.1021/ac60214a047`.